LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2023**

PST 2602 – MODERN PROBABILITY THEORY

Date: 06-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

Section A

Answer ALL questions **(10 X 2 = 20)**

1. Define decomposition of distribution functions.
2. Define probability measure.
3. How do you find limit of an increasing sequence of sets?
4. Define stability of independent random variables.
5. Define mixture of distribution functions.
6. State the properties of moment generating functions.
7. Define equivalent random variables.
8. Define bivariate characteristic function..
9. Define convergence in probability.
10. State Kolmogorov's strong law of large numbers.

Section B

Answer ANY FIVE questions **(5 X 8 = 40)**

11. State and prove the continuity property of probability.
12. State and prove the necessary and sufficient condition for n random variables to be independent.
13. Define convergence in probability and state and prove the criterion for convergence in probability
14. State and prove Jensens inquality.
15. Show that convergence in probability implies convergence in distribution.
16. Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$. Then prove that,

    (a) $a X_n \xrightarrow{P} a X$

    (b) $X_n + Y_n \xrightarrow{P} X + Y$

    (c) $X_n \, Y_n \xrightarrow{P} X \, Y$

    (d) $\dfrac{X_n}{Y_n} \xrightarrow{P} \dfrac{X}{Y}$

17. State and prove Kolmogorov's strong law of large numbers.
18. State and prove the conditions under which WLLN holds.

Section C

Answer ANY TWO questions. **(2 X 20 = 40)**

19. State and prove the necessary and sufficient condition for a function F to be the distribution function of a

random variable.

20. Define the characteristic function of a random variable and State the inversion theorem for discrete and continuous case and find

(a) Characteristic function φ (u) of normal distribution

(b) The distribution if $\varphi(u) = e^{-|t|}$, $-\infty < t < \infty$ (10 +10)

21. State and prove Markov's theorem.

22. State and prove the Lindeberg-Levi central limit theorem clearly explaining the assumptions.

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